

TJURINGOVE MAŠINE

ZA IZRAČUNAVANJE ARITMETIČKIH F-JA U AZBUCI $\Sigma = \{1\}$

1

1) OSNOVNE KRETNJE I PISANJA

l - MALA LEVA

0	*	L	1
0		L	1
1	*	H	1
1		H	1

r - MALA DESNA

0	*	R	1
0		R	1
1	*	H	1
1		H	1

x - BLANKO

0	*	*	1
0		*	1
1	*	H	1
1		H	1

l - REČKA

0	*		1
0			1
1	*	H	1
1		H	1

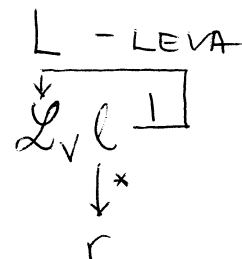
2) PRETRAŽIVANJA

L_V - VELIKA LEVA

0	*	L	1
0		L	1
1	*	H	1
1		L	1

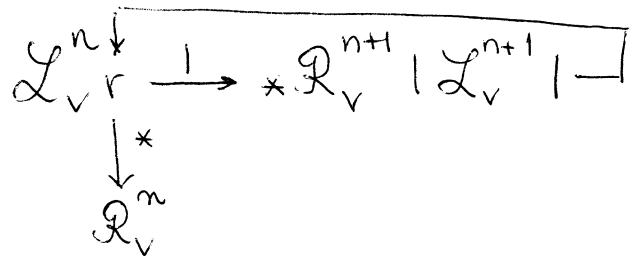
λ - LEVA ISTRAŽNA

0	*	L	1
0		L	1
1	*	L	1
1		H	1



Dualno L_V, g, R

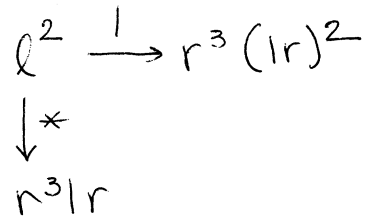
3) K_n - MAŠINA ZA KOPIRANJE : $*x_1* \dots *x_n*$ \longrightarrow $*x_1* \dots *x_n*x_i*$



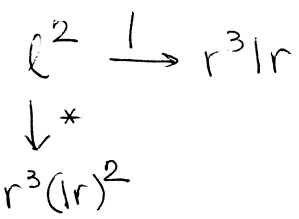
SABIRANJE: $\Sigma - *x*x*y*$ \longrightarrow $*x*x*y*x+y*$

$$K_2^2 \mathcal{L}_v | \mathcal{Q}_v (l^*)^2$$

Sg



$\bar{S}g$



4) MAŠINE ZA SPREMANJE

Del - BRISANJE : $\sim * x * \xrightarrow{\uparrow} \sim *$

\downarrow
 $\overbrace{L \xrightarrow{*} T}$

Dr - PRIVLAČENJE : $* \bar{x} * \dots * y * \xrightarrow{\uparrow} *$

\downarrow
 $\overbrace{L \xrightarrow{*} T}$

T - TRANSLACIJA : $\sim * x * \xrightarrow{\uparrow} \sim x *$

$r^2 \xrightarrow{L} l R_v l *$

$\downarrow *$
 l^2

S - SMICANJE ("SHIFT") : $\sim * x * y * \xrightarrow{\uparrow} \sim * y *$

\downarrow
 $\overbrace{L \xrightarrow{*} T}$

$\downarrow *$
 T

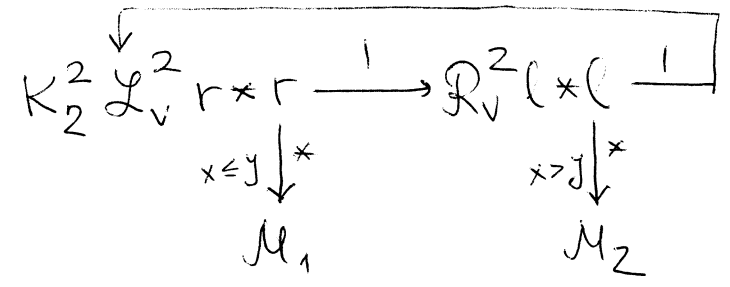
C - ČIŠČENJE : $\sim * \bar{x} * y * \xrightarrow{\uparrow} \sim y *$

($\bar{x} = x_1 * \dots * x_n$)

\downarrow
 $\overbrace{L \xrightarrow{*} R}$

$\downarrow *$
 $T L \xrightarrow{*} T$

5) OPŠTA POREDBENA MAŠINA



Abs

$\mathcal{M}_1: \mathcal{R}_V Dr$
 $\mathcal{M}_2: lr Dr$

Sub

$\mathcal{M}_1: \mathcal{R}_V Del \lambda r^2 lr$
 $\mathcal{M}_2: lr Dr$

Max

$\mathcal{M}_1: \mathcal{R}_V Del \lambda r K_1$
 $\mathcal{M}_2: Del \lambda r K_2$

Min

$\mathcal{M}_1: \mathcal{R}_V Del \lambda r K_2$
 $\mathcal{M}_2: Del \lambda r K_1$

Geg

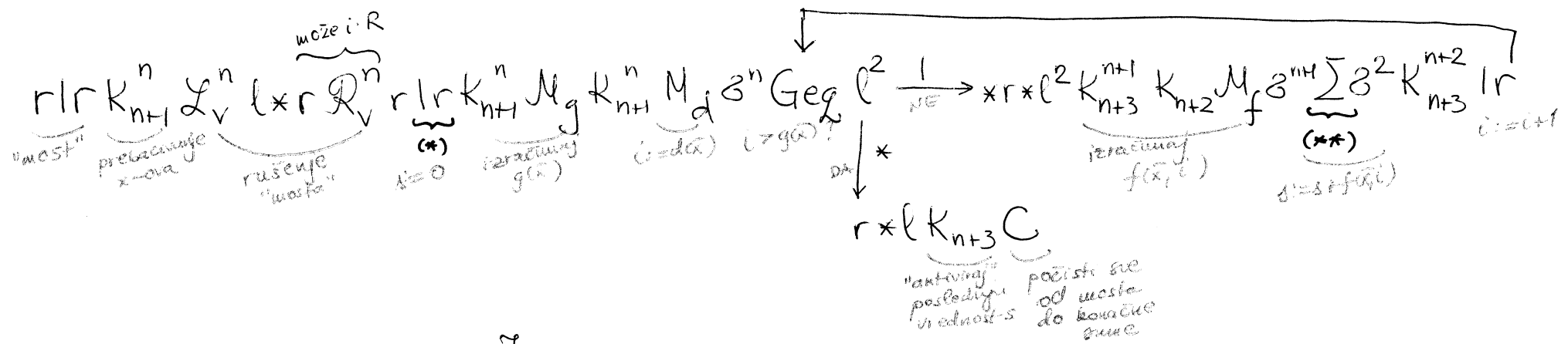
$\mathcal{M}_1: \mathcal{R}_V Dr l^2 \xrightarrow{1} r^2 Del r lr$
 \downarrow^*
 $r^2 lr$

$\mathcal{M}_2: Del \lambda r^2 (lr)^2$

Leg

$\mathcal{M}_1: \mathcal{R}_V Del \lambda r^2 (lr)^2$
 $\mathcal{M}_2: Del \lambda r^2 lr$

6) OPŠTA MAŠINA ZA SUMIRANJE: $h(x_1, \dots, x_n) = \sum_{i=d(\bar{x})}^{g(\bar{x})} f(x_1, \dots, x_n, i)$ (ako $d \leq g$ inače 0)



PROIZVOD Π : $xy = \sum_{i=1}^x x$. Dakle: $M_d: r(lr)^2$

$M_g: K_1$

$M_f: K_3$

7) OPŠTA MAŠINA ZA PROIZVOD: $h(x_1, \dots, x_n) = \prod_{i=d(\bar{x})}^{g(\bar{x})} f(x_1, \dots, x_n, i)$ (ako $d \leq g$ inače 1)

- isto kao šablon za sumu, sa modifikacijama

(*) : $(lr)^2$

(**) : Π

PRIMERI

• Exp : $x * y^x \rightarrow x * y * x^y$ $(0^0 = 1)$

$$x^y = \prod_{i=1}^y x$$

$$\left. \begin{array}{l} M_d : r(r)^2 \\ M_g : K_{-1} \\ M_f : K_3 \end{array} \right\} \rightarrow \text{tablon za proizvod}$$

• Fac : $x * x \rightarrow x * x!$

$$x! = \prod_{i=1}^x i$$

$$\left. \begin{array}{l} M_d : r(r)^2 \\ M_g : K_1 \\ M_f : K_1 \end{array} \right\} \rightarrow \text{tablon za proizvod}$$

• Δ : $x * y^x \rightarrow x * y * \lfloor \frac{x}{y} \rfloor$

$$\lfloor \frac{x}{y} \rfloor = \sum_{i=1}^x g_{eg}(x, yi)$$

$$\left. \begin{array}{l} M_d : r(r)^2 \\ M_g : K_2 \\ M_f : K_3 \prod \beta^2 G_{eg} \beta^2 \end{array} \right\}$$

• Rest : $\text{rest}(x, y) = x \div y \lfloor \frac{x}{y} \rfloor$

$\Delta \prod \beta K_3 K_2 \text{Sub} \beta^3$

• Div : $\text{div}(x, y) = \overline{sg}(\text{rest}(x, y))$

Rest $\overline{Sg} \beta$

8) SVAKA REKURZIVNA F-JA IMA SVOJU MAŠINU

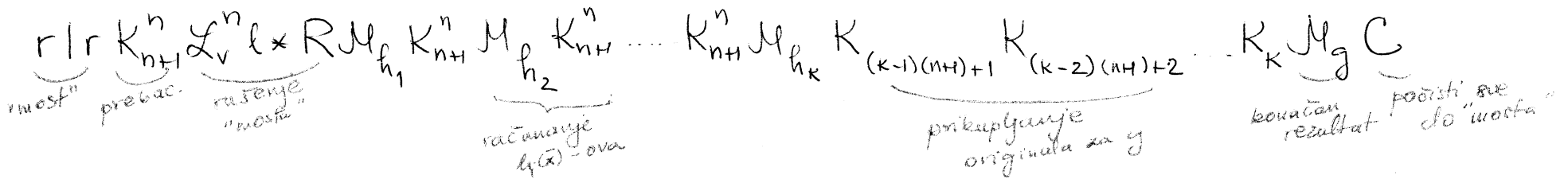
a) OSNOVNE F-JE

$N(x) = 0$ r | r

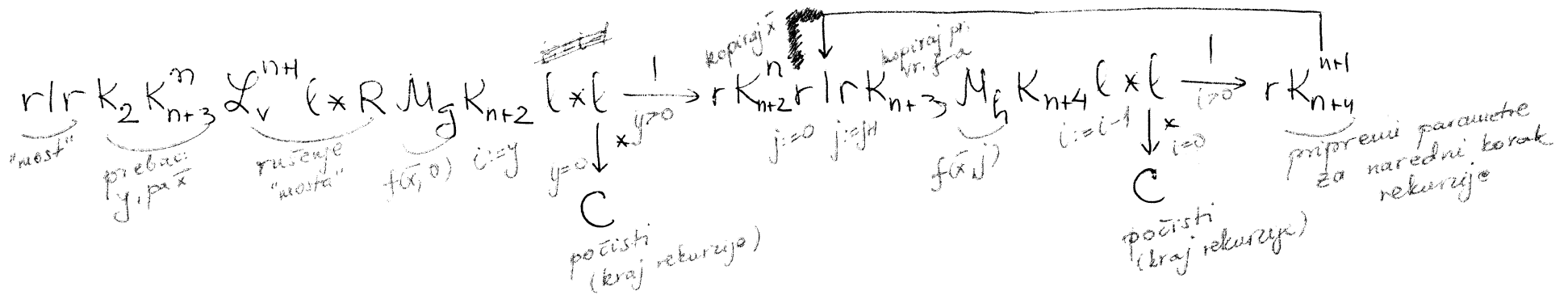
$S(x) = x + 1$ $K_1 | r$

$I_K^n(x_1, \dots, x_n) = x_K$ K_{n-K+1}

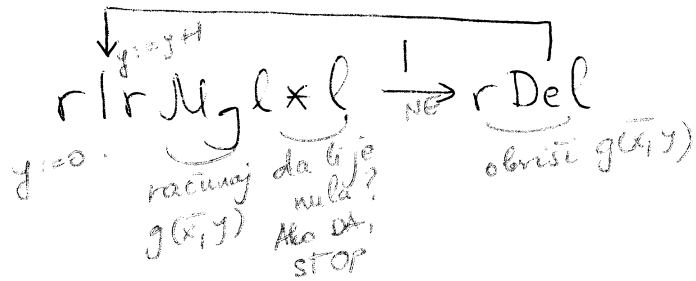
b) KOMPOZICIJA: $f(x_1, \dots, x_n) = g(h_1(x_1, \dots, x_n), \dots, h_k(x_1, \dots, x_n))$



c) ŠEMA PROSTE REKURZIJE: $f(x_1, \dots, x_n, 0) = g(x_1, \dots, x_n)$; $f(x_1, \dots, x_n, y+1) = h(x_1, \dots, x_n, y, f(x_1, \dots, x_n, y))$



d) MINIMALIZACIJA : $f(x_1, \dots, x_n) = \mu_y (g(x_1, \dots, x_n, y) = 0)$



PRIMER : $P - \begin{matrix} *x* \\ \uparrow \\ M_d: r(1r)^2 \end{matrix} \rightarrow *x* P_x * \uparrow$; $P? \begin{matrix} *x* \\ \uparrow \\ M_g: K_1 \end{matrix} \rightarrow *x* P_r(x) *$; $E: \begin{matrix} *x* y * \\ \uparrow \\ M_f: Div \end{matrix} \rightarrow *x* y * \exp_y x *$

$\mathcal{J}(x) = \sum_{i=1}^x \text{div}(x_i)$: $\left. \begin{matrix} M_d: r(1r)^2 \\ M_g: K_1 \\ M_f: Div \end{matrix} \right\} \rightarrow \text{šablon za sumu} \rightarrow P$

$\exp_y x = \mu_z (\text{div}(x, P(y)^{zH}) = 0)$
 $M_g: K_3^2 P K_3 1r \text{Exp} z^2 \text{Div} z^2$
 \downarrow
 šablon za minimalizaciju
 \downarrow
 E

$P_r(x) = \overline{sg} | \mathcal{J}(x) - 2 |$ $P?: \Gamma r(1r)^2 \text{Abs} z^2 \overline{Sg} z$
 $\mathcal{J}(x) = \sum_{i=1}^x P_r(x) - \left. \begin{matrix} M_d: r(1r)^2 \\ M_g: K_1 \\ M_f: P? \end{matrix} \right\} \rightarrow \text{šablon za sumu} \rightarrow P_i$

$P(x) = \mu_y (|\mathcal{J}(y) - (xH)| = 0)$
 $M_g: P_i K_3 1r \text{Abs} z^2 \rightarrow \text{šablon za minimalizaciju} \rightarrow P$